Chapter 8
Fatigue Strength and Endurance

Screen Titles

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Fatigue Strength Equation – 1, 2
Endurance Limit Behavior
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Chapter 8 begins the study of the behavior of mechanical elements subjected to fluctuating stress states that can produce fatigue failure. This chapter deals with the fatigue strength and endurance of materials. The subject is introduced by a generic description of the fatigue process. The subject of fatigue testing, cycle life behavior and endurance limit is considered in conjunction with estimating the endurance limit from the tensile strength of a material. Consideration is then given to factors that modify the endurance limit like surface condition, part size, load types, operating temperature and other miscellaneous effects. Several exercise problems and an extended sample problem is included to demonstrate the application of the subject content.

2. Page Index

Listed on this page are all the individual pages in Chapter 8 with the exception of the exercise problems. Each page title is hyperlinked to its specific page and can be accessed by clicking on the title. It is suggested that the reader first proceed through all pages sequentially. Clicking on the text button at the bottom of the page provides a pop up window with the text for that page. The text page is closed by clicking on the x in the top right corner of the frame. Clicking on the index button returns the presentation to the page index of chapter 8.
3. Fatigue Failure Process

Fatigue behavior in mechanical components is associated with the application of reversed or fluctuating stress states as contrasted to static loading behavior already studied. The number of reversed cycles of loading must be at least $10^3$ repetitions or greater for this behavior to become important. The associated failure mechanism in fatigue begins with the initiation of a small crack on the surface of the part. This may be due to some surface imperfection from processing particularly at sudden changes in geometry. The crack itself produces a high stress concentration that leads to continued growth of the crack.

4. Fatigue Failure Process (continued)

As the crack enlarges the remaining stress-bearing area of the part reduces in size and the applied stress levels increase. With additional cyclic loading this conditions worsens and failure of the part occurs. This failure is sudden and catastrophic. The broken surface has two appearances. Where the crack initiated and grew the surface appears shiny and almost hammered. The surface where the final fracture takes place displays the appearance of a brittle material that has failed in tension. The stress level at which fracture take place is generally significantly below the yield stress of the material. Hence fatigue is truly a different mechanism of failure from what is experienced by material pulled in tension through yielding on to the ultimate strength and final fracture.
5. Fatigue Strength Testing

To determine the fatigue strength of a material it is necessary to test the material to failure in a very different way from that used to determine its ultimate strength in tension. The technique employed makes use of a test specimen that is rotated at high speed subject to a pure reversed bending load. The dimensions and geometry of a common standard fatigue test specimen is shown in the figure with the loading forces in red. Tests to failure are conducted with such specimens on a Moore high-speed rotating beam machine at a number of increasing loads. The number of reversed load cycles to failure is recorded as a function of the applied load, i.e. maximum applied pure bending stress. Several tests are required at each load level to account for the statistical nature of the failure mechanism.

6. S-N Diagram

The results of the rotating beam specimen for steels behave generically as illustrated in the figure where fatigue strength in psi is plotted as a function of cycles to failure. This is referred to as an S-N diagram where S corresponds to stress and N corresponds to the number of cycles to failure. At one cycle or N equal to $10^0$ the fatigue strength is taken as equal to the ultimate strength in tension of the material. As the loading cycles increase the maximum stress level at which failure occurs decreases. Over the region of 1 to $10^3$ or 1000 cycles, referred to as low cycle fatigue, the reduction in the strength figure is very little. For N in the range of $10^3$ (one thousand) to $10^6$ (one million) cycles the reduction in the fatigue strength is much more significant. Repeated loading above $10^3$ cycles is considered high cycle fatigue. At some high cyclic level, usually around $10^6$ or $10^7$ load repetitions the S-N curve exhibits a distinct knee, becomes horizontal and the strength no longer decreases. This knee in the curve corresponds to the fact that a specimen at this stress level will effectively exhibit infinite life. The level of fatigue strength associated with this phenomenon is referred to as the endurance limit of the material and is measured as a stress in psi. Effectively all ferrous material and alloys exhibit this endurance limit behavior but non-ferrous materials do not. Their fatigue strength continues to decrease beyond $10^6$ cycles but normally at a reduced rate.
7. Exercise Problem -1

It is somewhat difficult to conceptualize the true difference between $10^3$ and $10^6$ events, which have great significance in fatigue, without attaching some physical significance to such events in terms of every day experiences. The two exercises presented on this page will help accomplish this. Work out both of them and check your answer by clicking on the solution button before going on.

(Solution on Page 187)

8. Fatigue Strength Equation

To carry out numerical analyses and predictions of fatigue behavior it is necessary to adopt some mathematical model to represent the generic behavior depicted on the S-N diagram previously described. For $10^3$ to $10^6$ cycles of load repetition, which is considered the most important region in high cycle finite fatigue life, the relation of fatigue strength to cycles to failure is assumed to be a straight line on a log-log plot. That is, the fatigue strength, $\sigma_f$, is represented as a constant “a” times $N$, the cycles to failure, raised to some power “b”. The numerical values of “a” and “b” are dependent on the specific properties of the material in question. To determine “a” and “b” two conditions are assumed. The first is that at $10^3$ cycles the fatigue strength, $\sigma_f$, is assumed to be equal to 90% of the ultimate tensile strength in tension of the material. This is consistent with the S-N diagram behavior where the decrease in fatigue strength from the ultimate strength is quite small in the low cycle region. The second condition is that at $10^6$ cycles the fatigue strength will be equal to the endurance limit of the material designated as $\sigma_e$. The log of both sides of the assumed fatigue behavior equation are now taken and the conditions representing the two ends of this line are substituted in to give equations 1 and 2 at the bottom of the page.
Fatigue Strength Equation (cont.)

Equation 2 is now subtracted from two times equation 1. This eliminates $b$ from the resulting equation permitting “$a$” to be determined. It is seen that the final result for “$a$” is dependent on the tensile strength and the endurance limit of the material in question. To determine “$b$” equation 2 is again subtracted directly from equation 1 to eliminate “$a$”. Solving for “$b$” gives the result shown that is again dependent on the tensile strength and the endurance limit of the material. The final result at the bottom of the page can be used to determine $\sigma_f$ for any $N$ between $10^3$ and $10^6$ cycles or conversely if given a specific fatigue stress to which the material is subjected it finite life in cycles can be determined. It should be noted here that the minimum ultimate strength in tension $\sigma_u$ should always be used in this equation for fatigue strength.

9. Fatigue Strength Equation (cont.)

10. Endurance Limit Behavior

Since endurance limit values are not easily obtainable it is both desirable and convenient to have some means of relating the endurance limit to the ultimate tensile strength of the material since data on ultimate strength is usually more readily available. It might be anticipated or at least hoped for that the endurance limit might be somehow directly related to ultimate strength. As the ultimate tensile strength increases it seems reasonable that endurance limit might increase also. Consider a plot of endurance limit test results versus tensile strength for a variety of ferrous materials and alloys. The graph shown here depicts three possible linear relationships where the ratio of tensile strength to endurance limit is 0.4, 0.5 and 0.6. Tensile and rotary beam tests on a variety of wrought iron specimens gives the blue cluster of results as shown on the graph. Similarly, test results from a variety of carbon steels gives the two red clusters depicted on the graph. Finally, tests on a variety of alloy steels gives the two green clusters of results on the graph. It is observed that up to a tensile strength of 200 kpsi the blue, red and green clusters seem to average out close to the $\sigma_e/\sigma_u$ line of 0.5. Above a tensile strength of 200 kpsi the results from both the carbon and alloy steels appear to flatten out quite appreciably. Based on these results a conservative model has been adopted and is generally used for estimating the endurance limit of a ferrous material based on its ultimate tensile strength.
11. Endurance Limit Equations

From statistical considerations of a large number of test results of ferrous materials it has been determined that for materials with ultimate tensile strengths below 200 kpsi a reasonable assumption for estimating their endurance limit is given by the equation \( \sigma_e = 0.504 \sigma_u \) for \( \sigma_u \leq 200 \) kpsi

\[ \sigma_e = 100 \text{ kpsi} \] for \( \sigma_u > 200 \) kpsi

where \( \sigma_u = \text{minimum tensile strength} \)

and \( \sigma_e = \text{rotating beam specimen} \)

12. Exercise Problem – 2

This exercise provides an opportunity to apply the material and models just covered to estimating endurance limit, the fatigue strength for finite life and the life associated with a given application of fatigue loading. When finished with your analysis click on the solution button to check your numerical results. Then proceed on to the next page in the chapter.

(Solution on Page 188)
### Modifying Factors

\[ \sigma' = k_1 k_2 k_3 \sigma_0 \]

where \( \sigma' \) = endurance limit of part  
\( \sigma_0 \) = endurance limit of test specimen  
\( k_1 \) = surface factor  
\( k_2 \) = size factor  
\( k_3 \) = load factor  
\( k_4 \) = temperature factor  
\( k_5 \) = miscellaneous effects factor

#### 13. Modifying Factors

The endurance limit discussed so far has been the test value either obtained directly from rotating beam tests or estimated as such from the tensile strength of the material. This value of endurance limit is in almost every instance higher than that of an actual part due to a number of factors not included in the beam test conducted under ideal standard conditions. This is represented analytically by multiplying the rotating beam endurance limit by a number of \( k \) factors to give the endurance limit of the part. To begin with a rotating beam specimen is polished longitudinally to reduce any surface imperfection that might initiate a crack. The rougher condition of processed real part surfaces will always reduce the value of the endurance limit. Other factors that affect the actual endurance limit include the size of the part, the type of loading it is subjected to, the temperature at which it operates and other miscellaneous conditions including stress concentration factors due to changes in geometry. Each of these factors will now be examined and treated separately.

#### 14. Surface Factor

The surface condition factor is represented analytically by the equation \( k_a \) is equal to a constant “a” times the tensile strength raised to the “b” power. Values of “a” and “b” for different processed part surface conditions are indicated in the included table. As might be expected the value of \( k_a \) is always less than one and decreases in value proceeding from a smooth ground surface finish to a rough forged part surface. The condition of the part surface has one of the greatest impacts on the reduction of the ideal endurance limit value.

<table>
<thead>
<tr>
<th>Surface Finish</th>
<th>Factor “a” (ksi)</th>
<th>Exponent “b”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>1.34</td>
<td>-0.006</td>
</tr>
<tr>
<td>Machined</td>
<td>2.70</td>
<td>-0.266</td>
</tr>
<tr>
<td>Cold Drawn</td>
<td>3.70</td>
<td>-0.266</td>
</tr>
<tr>
<td>Heat Treated</td>
<td>4.4</td>
<td>-0.718</td>
</tr>
<tr>
<td>Forged</td>
<td>19.0</td>
<td>-0.906</td>
</tr>
</tbody>
</table>
15. Surface Factor Impact

To illustrate the effect of the surface factor consider the value of \( k_a \) for a part with a machined surface and a tensile strength of 65 kpsi. Applying the formula and values of “a” and “b” from the previous page gives a value of \( k_a \) of 0.894 or an 11% reduction in the endurance limit. If the same part were forged the \( k_a \) value would drop to 0.627 or a 37% reduction in \( \sigma_e \). From the values in the table it is seen that increasing the tensile strength of the material reduces \( k_a \) even further to the point that a forged part with a tensile strength of 125 kpsi reduces the endurance limit of the material by more than 65%. As mentioned previously surface condition has one of the largest effects on endurance limit. Why do you think this is the case?

16. Size factor

Somewhat surprising is that the overall size of the part can also affect its endurance limit. For a solid rotating shaft up to 2 inches in diameter the size factor \( K_b \) is given by the expression the quantity the diameter divided by 0.3 raised to the \(-0.1133\) power. Again the \( k_b \) factor will be less than one. For solid-rotating shafts greater than 2 inches in diameter a value of \( k_b \) between 0.6 and 0.7 should be used with the smaller value employed for a more conservative design. For hollow or non-rotating shafts an effective diameter equal to 0.370 of the outside diameter should be used in the previous equation. For a rectangular cross section the effective diameter is given by 0.808 times the base multiplied by the height with the product raised to the 0.5 power. For axial loads there is no size factor effect so that \( k_b \) is one.
17. Size Factor Impact

The numerical effect of the size factor is demonstrated by calculating $k_b$ for a rotating solid shaft 2 inches in diameter subjected to a bending load. Substituting 2 inches into the $k_b$ formula gives a result of 0.81 or a 19% reduction in endurance limit by itself. However, if coupled with a machined surface part for which $K_a$ is 0.89 the combined effect is a multiplier of 0.72 or a 28% reduction in endurance limit. Now compare the value of $k_b$ for the solid shaft with a hollow shaft. The effective diameter is 0.74 giving a value of $K_b$ of 0.90, which is higher than for the solid shaft. Of course the stress for a given bending load would also be higher in the hollow shaft. Finally consider a rotating solid shaft with a diameter of .5 inches. In this case the size factor calculates out to be 0.94 or only a 6% reduction.

18. Load Factor

This factor has one of the least effects on the endurance limit of the part except for the case of pure torsion. The value of the factor for axial load is dependent on the tensile strength of the material. For materials with tensile strengths below 220 kpsi the value of $K_c$ is taken to be 0.923. For materials with tensile strengths above 220kpsi the value of $K_c$ is unity. For bending loads $K_c$ is taken as unity for all values of tensile strength. For torsion or direct shear the $K_c$ value is given as 0.577 for all tensile strengths. Does this number look familiar? How about the distortion energy theory?
19. Temperature Factor

The behavior of endurance limit with operating temperature of the part is assumed to be similar to that of the tensile strength of the material. Contrary to what might be expected, tensile strength actually increases slightly with operating temperature compared to room values up to between 400 and 500 degrees F. The value of $k_d$ for the temperature effect on endurance limit is taken to be equal to the ratio of tensile strength at operating temperature to tensile strength at room temperature as given in the table on this page. Note that the reduction impact of this factor does not become very significant until operating temperatures are above 600 degree F.

### Temperature Factor

$$k_d = \frac{\sigma_T}{\sigma_{RT}}$$

<table>
<thead>
<tr>
<th>Temp. °F</th>
<th>$\sigma_T/\sigma_{RT}$</th>
<th>Temp. °F</th>
<th>$\sigma_T/\sigma_{RT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.000</td>
<td>600</td>
<td>0.925</td>
</tr>
<tr>
<td>200</td>
<td>1.000</td>
<td>700</td>
<td>0.937</td>
</tr>
<tr>
<td>300</td>
<td>1.024</td>
<td>800</td>
<td>0.972</td>
</tr>
<tr>
<td>400</td>
<td>1.018</td>
<td>900</td>
<td>0.937</td>
</tr>
<tr>
<td>500</td>
<td>0.995</td>
<td>1000</td>
<td>0.950</td>
</tr>
</tbody>
</table>

20. Miscellaneous Effects Factor

Within this category are grouped effects that include residual stress, corrosion, electrolytic plating, metal spraying, cyclic frequency, fretting corrosion, and stress concentrations. Compressive residual stresses increase fatigue strength. This accounts for why shot peening, surface hammering, and cold working of surfaces are employed. Corrosion tends to reduce fatigue strength as it leads to a rougher surface on which cracks can more easily initiate. Plating can reduce fatigue life by as much as 50% while the reduction effect of metal spraying is more like 15%. The effect of frequency changes or the rate of loading is usually considered minimal but may take on more importance at elevated temperatures and with corrosion. Frettage corrosion due to microscopic movement at tight joints like press fits or bolted connections can have an appreciable effect but is extremely hard to quantify. Finally, stress concentrations introduced by sudden changes in part geometry and shape will also reduce fatigue strength and can be handled more quantitatively than the other miscellaneous factors. This will be treated in more detail on the next page.
21. Stress Concentrations

For low cycle fatigue or static loads, stress concentration effects can generally be neglected unless the part material is brittle. For cyclic loading above $10^6$ cycles the factor $k_e$ is expressed as the reciprocal of the geometric stress concentration factor $K_f$. The appropriate values of $K_f$ must be obtained from references dealing specifically with the topic of stress risers due to changes in part geometry and the type of applied loading. This will not be dealt with here. For cyclic loading between $10^3$ and $10^6$ cycles the $K_f$ prime factor is given by $a$ times the tensile strength raised to the $b$ power where $a$ is one over $K_f$ and $b$ is minus one third of the log of one over $K_f$. This is similar to the model used for calculating finite life fatigue strength between a thousand and a million cycles of loading.

Example Problem

A 3/4 in. diameter shaft that will operate at 1800 rpm under a constant bending load is machined from the steel in Exercise Problem 2, i.e., ultimate strength of 97 ksi and yield stress of 75 ksi. The shaft will operate at room temperature and contains a radial hole that introduces a stress riser of 1.5. Estimate the values of the modifying factors and determine the endurance limit for the part for the conditions specified.

Example Problem

All of the previous subject content will now be applied to a realistic example problem. A 3/4 inch diameter shaft that will be operate at 1800 rpm under a constant bending load is machined from steel with the properties given in Exercise Problem 2, that is the tensile strength is 97 ksi and the yield strength is 75 ksi. The shaft will operate at room temperature. It contains a radial hole that introduces a stress riser of 1.5. Estimate the values of the modifying factors and determine the endurance limit for the part for the condition specified. This problem will be worked out numerically on the next three pages.
23. Problem Solution - 1

The solution is begun by estimating the rotating beam specimen endurance limit sigma e. Since the tensile strength is less than 100 kpsi the value of sigma e is obtained by multiplying the tensile strength by .504 to give 48.8 kpsi. Next the modifying factors are each calculated for the given problem conditions. With the surface of the shaft being machined the surface factor ka is given by 2.7 times the tensile strength 97 raised to the −0.265 power. This gives a ka value of 0.802.

24. Problem Solution - 2

With the problem specifying that the shaft is rotating and subject to a bending load the size factor kb is given by the shaft diameter of .75 inches divided by 0.3 with the quotient raised to the −0.113 power. This gives kb a value of .902. Since the loading is bending and the operation is at room temp both the loading factor, kc, and the temperature factor kd are equal to 1.
25. Problem Solution - 3

The stress concentration factor $k_e$ is given by the reciprocal of the geometric stress concentration factor of 1.5 due to the hole through the shaft. This gives a value for $K_e$ of 0.667. The final value for the modified endurance limit is then the product of all the modifying factors times the estimated endurance limit of the rotating beam specimen. Carrying out these multiplications gives a value of 23.7 kpsi for the endurance limit of the problem shaft. Note that this is a reduction of more than 50% of the estimated test specimen material property. This illustrates quite clearly how the geometric, loading and operating conditions of the part can significantly impact its finished fatigue strength.

26. Review Exercise

In this review exercise you are to type in answers from the keyboard into the blank spaces. Clicking on the tab key will provide immediate feed. If the answer is incorrect another response can be entered. A subsequent click on the tab key will move the cursor to the next blank. Hot words are provided in each question to revisit the page in the chapter on which the correct answer can be found. After completing the exercise click on the next page button.
27. Off Line Exercise

A rotating cantilevered stepped shaft supported by two bearings carries a load of 175 lbs. as indicated. The shaft surface is machined and ground from a steel with $\sigma_y = 95$ kpsi and $\sigma_t = 105$ kpsi. The step is radiused introducing a stress concentration factor of 1.30 and the part will operate at room temperature. Estimate the life of the part.

(Solution in Appendix)
Chapter 8

Fatigue Strength and Endurance

Problem Solutions

Screen Titles

Problem 1 Solution
Problem 1 Solution (cont.)
Problem 2 Solution
Problem 2 Solution (cont.)
1. Problem 1 - Solution

To determine the distance traveled by a car in miles per million revolutions of its engine begin with the average speed of the car, 30 mph. Then multiply by the reciprocal of the minutes per hour and the reciprocal of the revolutions per min. This gives the units of miles per revolution on the right side of the equation. Multiplying both sides by ten to the sixth revolutions gives a final answer of 200 miles. A rather significant distance at this average speed. Contrast this with the distance associated with a thousand cycles that gives a distance of .2 of mile or just slightly more than a thousand feet. Similarly the time required to travel the 200 miles is obtained by dividing the distance by the average speed of 30 mph giving 6.7 hours while the time required to travel .2 miles is less than half a minute. Not surprising since this corresponds to 1000 cycles of an engine turning over at 2500 rpm.

2. Problem 1 Solution (cont.)

For a clock ticking one cycle a second the time required for a million ticks is obtained by multiplying one cycle per second times the reciprocal of 60 sec per min times the reciprocal of 60 min per hour times the reciprocal of 24 hr per day all multiplied by ten to the sixth clicks. This gives the result of 11.57 days. In contrast the time required for a thousand ticks is just over one quarter of an hour. The number of ticks in a year is calculated to be over 30 million. Hopefully, these examples provide a better concept of how small 10^3 events is compared to 10^6 repeated occurrences particularly as it applies to design for fatigue strength and endurance. When you have finished with this solution click on the return button to go to the next page in Chapter 8.
3. Problem 2 - Solution

Since the tensile strength is less than 200 kpsi then for part a the estimate of the endurance limit is simply given by .504 times the tensile strength. This gives an endurance limit of 48.8 kpsi. To calculate the fatigue strength at 10⁵ cycles the factors “a” and “b” must first be determined using the given tensile strength and estimated endurance limit. Carrying out the required calculations gives “a” equal to 156 kpsi and “b” equal to minus .084. Substituting these values into the equation for fatigue strength at a finite life gives a final value of 59.3 kpsi for sigma f. Note that this value lies between sigma e and sigma u as it should.

4. Problem 2 – Solution (cont.)

The finite life fatigue strength equation is again used to determine the life in cycles to failure for an applied reversed stress loading of 63 kpsi. The values of “a” and “b” remain the same as on the previous page. Solving for N with sigma f specified results in a finite life value of 48 time 10⁴ cycles. When you have finished with this solution click on the return button to go to the next page in Chapter 8.